EXAMPLE PROOF TO HELP WITH PROBLEM #13 of Sec. 8.2

Sec 8.3, #19 (NOT ASSIGNED)

Define the relation F on $\mathbb{Z}$ by requiring that, for all $m, n \in \mathbb{Z}$,

$$m F n \Leftrightarrow 4 \mid (m-n).$$

To Prove: F is an equivalence relation

Proof: [F is reflexive]

Let x be any integer. Then, $x - x = 0 = 4 \times 0$.
$\therefore 4 \mid (x-x)$. $\therefore xFx$, by def'n of relation F.
$\therefore$ F is reflexive, by direct proof.

[F is symmetric]

Let x and y be any integers. Suppose xFy. [NTS: yFx].
Then, $4 \mid (x-y)$, by def'n of relation F. $\therefore x - y = 4k$ for some integer k.
$\therefore (y-x) = (-1)(x-y) = (-1)(4k) = 4(-k)$.
$\therefore 4 \mid (y-x)$. $\therefore yFx$, by def'n of relation F.
$\therefore$ F is symmetric, by direct proof.

[F is transitive] Let x, y and z be integers such that xFy and yFz.
[NTS: xFz]. $\therefore$ By def'n of F, $4 \mid (x-y)$ and $4 \mid (y-z)$.
$\therefore$ There exist integers k and l such that $(x-y) = 4k$ and $(y-z) = 4l$. Now, $(x-z) = (x-y) + (y-z)$.
$\therefore x - z = 4k + 4l$, by substitution, $\therefore x - z = 4(k+l)$.
$\therefore 4 \mid (x-z)$. $\therefore xFz$, by def'n of relation F.
$\therefore$ F is transitive by direct proof.

$\therefore$ F is reflexive, symmetric and transitive, $\therefore$ F is an equivalence relation. QED